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MODEL: Multi-Objective Differential Evolution with Leadership Enhancement

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***Abstract* — Differential Evolution (DE) has been successfully used to solve various complex optimization problems; however, it can suffer depending of the complexity of the problem from slow convergence due to its iterative process. The use of the leadership concept was efficiently utilized for the acceleration of Particle Swarm Optimization (PSO) in a single-objective space. The generalization of the leadership concept in multi-objective space is not trivial. Furthermore, despite the efficiency of using the leadership concept, a limited number of multi-objective metaheuristics utilize it. To address these challenges, this paper incorporates the concept of leadership in a multi-objective variant of DE by introducing it into the mutation scheme. The preliminary results are promising as MODEL outperformed the parent algorithm GDE3 and showed the highest accuracy when compared with seven other algorithms.**

***Keywords* — Multi-objective optimization, leadership, differential evolution, DE, metaheuristics, evolutionary algorithms.**

I. Introduction

The DE algorithm is an efficient stochastic method to solve complex real-life global real optimization problems which can be non-convex, multimodal, non-differentiable, and discontinuous to name but a few. DE has been extensively studied for single-objective and multi-objective optimization problems [1]. However, DE suffers, as most metaheuristics, from slow convergence depending of the complexity of the problem.

Several attempts of accelerating the multi-objective variant of DE have been reported since its first invention in 1999 by Chang et al. [2]. Later, the Pareto Differential Evolution (PDE) has been proposed by Abbas and Sarkar [3] which limited the reproduction process among only the non-dominated solutions. Then, the first version of the Generalized Differential Evolution (GDE) [4] has been proposed by Kukkonen and Lampinen where the selection process of GDE was based on Pareto dominance, in GDE2 [5] a crowding distance was utilized to select best solutions, and in GDE3 [6] a growing population size and non-dominance sorting were utilized to select best solutions. Despite its accuracy, GDE3 appeared to be slower than MO variants of PSO for certain kinds of objectives [7].

One reason of PSO being sometimes faster than DE is probably the use leadership concept. In this paper, it proposed to further enhance GDE3 by incorporating the use of leadership into GDE3.The selection of leader is based on the non-dominance sorting but the number of possible leaders, called deputies, to be selected is restricted to preserve their well-distribution. The proposed contribution consists in modifying the mutation scheme as follows:

- Incorporate leadership into the mutation scheme using deputies: GDE3/deputy/1
- Implementation of the non-dominance sorting in the difference vector.
- Utilization of opposite control of parameter F with a certain probability to avoid premature convergence in certain cases.

The reminder of this paper is organized as follows. Section II introduces briefly to multi-objective optimization. Section III describes the proposed MODEL algorithm. Section IV provides details about the parameter settings. Section V presents the results of the experiments. Finally, Section VI concludes the paper.

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II. Multi-Objective Optimization

A MOO problem consists of multiple objectives to be simultaneously minimized or maximized. However, for the sake of simplicity and without any loss of generality, in this paper is assumed that all the objective functions are to be minimized. Also, a MOO problem can be subject to equality and inequality constrains and variable boundaries. It is formulated as follows:

$$Min\ F(x) = [f_1(x), f_2(x), \dots, f_k(x)] \quad (1)$$
$$s.t.$$
$$g_i(x) \geq 0, \ \ i = 1, 2, \dots, m \quad (2)$$
$$h_i(x) = 0, \ \ i = 1, 2, \dots, p \quad (3)$$

where x is a vector of decision variables for the optimization problem, $f_i(x)$ are objective functions, $g_i(x)$ are inequality constraints, and $h_i(x)$ are equality constraints. The equations (2) and (3) determine the *feasible region* $\Omega \subseteq \mathbb{R}^n$.

To facilitate the understanding of the paper, some definitions related to multi-objective optimization are provided such as Pareto dominance, Pareto weak dominance, Pareto optimality, Pareto optimal set, and Pareto optimal front described below.

Definition 1: Weak Pareto dominance

Given two solutions $x, y \in \Omega$, it is said x weakly *dominates* y (denoted by $x \preccurlyeq y$) iff $\forall\, i = 1, \dots, k\ f_i(x) \leq f_i(y)$, i.e., a solution x Pareto weakly dominates a solution y, if x is at least as good as y in every objective.

Definition 2: Pareto dominance

Given two solutions $x, y \in \Omega$, it is said x *dominates* y (denoted by $x \prec y$) iff $\forall\, i = 1, \dots, k\ f_i(x) \leq f_i(y)$ and $\exists\, i = 1, \dots, k$ where $f_i(x) < f_i(y)$, i.e., a solution x Pareto dominates a solution y, if x is at least as good as y in every objective and better than y in at least one objective. If x and y do not dominate each other, it said they incomparable (x || y)

Definition 3: Pareto optimality

Given a solution $x^* \in \Omega$, x^* is said to be *Pareto Optimal* if $\nexists$ $y \in \Omega$ such that $y \prec x^*$.
x^* is Pareto optimal if it is not dominated by any other solution.

Definition 4: Pareto optimal set

A Pareto optimal set is defined by $PS = \{x \in \Omega \mid x \text{ is a Pareto optimal solution}\}$
i.e., a Pareto set PS is a combination of non-dominated solutions.

Definition 5: Pareto optimal front

A Pareto optimal front is defined by $PF^* = \{F(x) = [f_1(x), f_2(x), \dots, f_k(x)] \mid x \in \text{PS}\}$
i.e., the non-dominated solutions make up a Pareto-optimal front when visualized in the objective space.

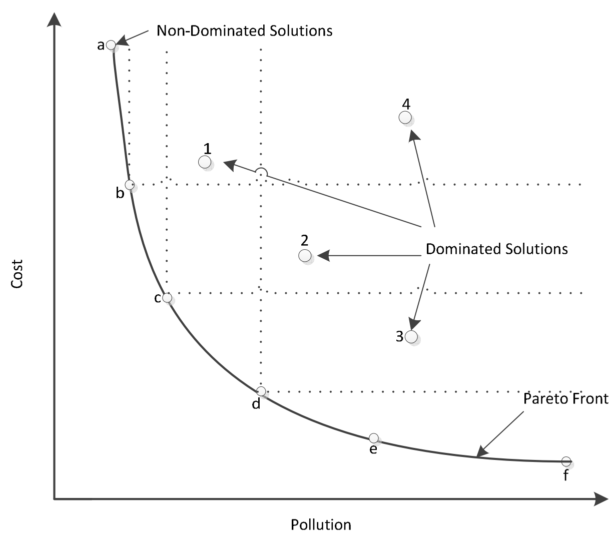

Figure 1: Pareto Front Schematic. Bi-objective minimization problem with cost and pollution objectives

As shown in Figure 1, for example, an energy system design might involve two objectives, such as cost and pollution. One solution might be inexpensive but very polluting such as solution *f*, while another solution can be affordable but more polluting such as solutions *c, d,* or *e*. A last solution can be very expensive but with almost no pollution such as solution *a*. The optimal solutions which are *non-dominated* (*a,b,c,d,e,f*) represent the *Pareto optimal set* which when viewed together in the objective space form the *Pareto front*.

III. PROPOSED MUTATION SCHEME IN MODEL

The general convention used to represent mutation schemes in DE is DE/x/y/z, where x stands for the base vector to be perturbed, y is the number of difference vectors used for one perturbation of x, and z represents the type of crossover being used (exp: exponential, bin: binomial) [8].

The classical scheme in the single objective version of DE is DE/rand/1 [9]. "rand" means the three vectors used for mutation are randomly selected from population, "1" means there is only one difference vector. So, the mutation is done as follows:

$$\vec{V_t} = \vec{X}_a + F \times (\vec{X}_c - \vec{X}_b), \quad (1)$$

where $\vec{X}_a$, $\vec{X}_b$, $\vec{X}_c$ are randomly selected vectors from the current population and $i \neq a \neq b \neq c$, i is the current vector. $F \in [0, 2]$ is a real constant to determine the amplification of the differential variation $(\vec{X}_c - \vec{X}_b)$.

This section presents the three contributions made to the classical mutation scheme of GDE3 which are the use of GDE3/deputy/1 scheme, the utilization of the non-dominance ranking in the difference vector, and the sign change of the factor *F* with a certain probability.

A. Use of leadership in the mutation (GDE3/best/1)

In single objective optimization, the DE/best/1 scheme works as follows:

$$\vec{V_t} = \vec{X}_{best} + F \times (\vec{X}_c - \vec{X}_b), \quad (2)$$

where $\vec{X}_{best}$ is the fittest found solution.

The purpose of using the fittest solution is to achieve a faster convergence of the population than in PSO. The problem is that there is no best (fittest) solution in MOPs. A common and logical alternative is the random selection of one of the non-dominated solutions as in SMPSO [10]. However, it has been shown recently if non-dominated solutions are located in the same area in the objective space, then they will pull the entire population towards that specific region which affects the well-distribution of the Pareto optimal solutions [11]. Consequently, the population will end either by a premature convergence or a slower convergence.

As a solution, it is proposed to select more representative leaders, called deputy leaders, in a more representative (well-distributed) manner than in [11]. So, it is proposed to use the GDE3/deputy/1 scheme where deputies are selected among leaders to better represent the population. This proposed scheme is the main cause of GDE3 performance improvement. But, the deputy leaders are selected differently than in Leader and Speed constraint Multiobjective PSO (LSMPSO) [11] which used the Weighted Sum Method for leader selection. Actually, p% of deputy leaders are selected using the classical non-dominance sorting which uses the non-dominance and crowding distance discriminants as follows:

$$\vec{V_t} = \vec{X}_{deputy} + F \times (\vec{X}_c - \vec{X}_b), \quad (3)$$

where $\vec{X}_{deputy}$ is a p% more representative solutions of the population which are selected using the non-dominance sorting and crowding as secondary discriminant.

Restricting the number of best (non-dominated) to $p\%$ serves to maintain a good-distribution among deputy leaders using the crowding distance mechanism and avoid premature convergence or a slowdown in convergence speed due to a concentration of leaders in specific region of the objective space. In this paper, the parameter p is set to 0.1 (i.e., p = 10%) similar to [11].

B. *Non-dominance sorting for the difference vector*

For further acceleration of the convergence speed; it is proposed to apply the non-dominance sorting between the vectors used to generate the difference vector. Intuitively, the difference vector should be, as in PSO, from a better solution to a lower quality solution to converge towards an optimum. Assume two random candidate solutions c and b from the population to form the difference vectors; if c dominates b, then the difference vector is $\vec{X}_c - \vec{X}_b$. If both solutions are incomparable, then the less crowded solution is selected based on the crowding distance using in NSGA-II [12]. The proposed mutation scheme is described below. The bold sections are the modified portions of the difference vector.

Proposed (in bold) Difference Vector

If $\left(\left(\boldsymbol{\vec{X}_c \prec \vec{X}_b}\right) \textbf{ OR } \left(\left(\boldsymbol{\vec{X}_c \,||\, \vec{X}_b}\right) \textbf{ AND } \left(\boldsymbol{CD}(\boldsymbol{\vec{X}_c}) < \boldsymbol{CD}(\boldsymbol{\vec{X}_b})\right)\right)\right)$

$\vec{V_t} = \vec{X}_{deputy} + F \times \left(\boldsymbol{\vec{X}_c - \vec{X}_b}\right)$

Else

$\vec{V_t} = \vec{X}_{deputy} + F \times \left(\boldsymbol{\vec{X}_b - \vec{X}_c}\right)$

EndIf

where $\vec{X}_{deputy}$ is randomly selected among deputy leaders, and $\vec{X}_c$ and $\vec{X}_b$ are randomly selected from the population, deputy $\neq a \neq b$.

CD: crowding distance.

C. *Opposite of Control Parameter F*

The earlier proposed mutation schemes, namely, the utilization of deputies and the non-dominance ranking in difference vector served to increase the convergence speed while preserving a uniform distribution of the population of DE [13]. However, our observations show in specific cases these modifications can lead to premature convergence. For example, for the problem DTLZ5, premature convergences might happen because the search space is very large, i.e. there is a large distance between the initial population and the true Pareto optimal solutions in the objective space as illustrated in Figs 2 and 3. Fig. 2 shows a problem where the initial distance between the population and the true PF is reasonable whereas the Fig. 3 shows a case where the initial distance between the population and the true PF is large enough to cause premature convergence due the use of leadership in the mutation scheme of GDE3.

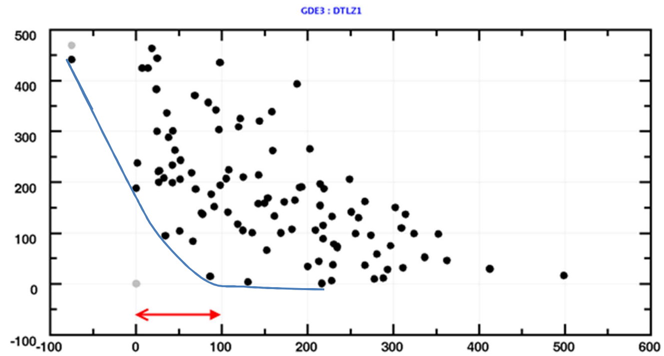

Fig. 2: Example of a problem where the distance in the objective space between the initial population and the PF is reasonable

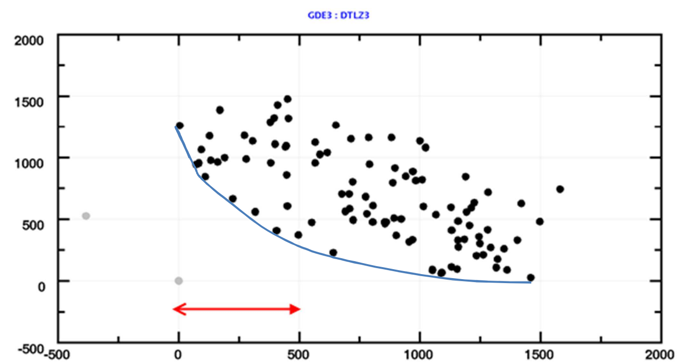

Fig. 3: Example of a problem where the distance in the objective space between the initial population and the PF is large

To solve this problem of premature convergence, it is proposed to utilize a negative control parameter F with a certain probability which is fixed to 0.3 in our experiments similar to the jumping rate used in the ODE [14]. Changing the sign of F serves to relax the proposed utilization of the non-dominance ranking in the difference vector and avoid a premature convergence by strengthening the explorative properties of MODEL. In other words, sometimes the movement of a candidate solution is directed in an opposite direction to the optimum to escape from local optima. A similar approach has been used in [15] for single objective problems.

D. *Multi-Objective Differential Evolution with Leadership*

By integrating the three proposed mutation modifications, presented in previous sub-sections, into GDE3, a new MO metaheuristic (MOM) is proposed, called Multi-objective Differential Evolution with Leadership (MODEL). The details of the three proposals to build a new mutation operator of GDE3 to accelerate its convergence speed are described in the pseudocode provided below.

MODEL Algorithm. In bold, the modification added to GDE3.

1. Evaluate the initial population P of random individuals.
2. While stopping criterion not met, do:
   2.1. For each individual P_i ($i = 1, \ldots, popSize$) from P repeat:
      (a) Generate candidate t from parent P_i as follows.

$$\textbf{If}\left(\left(\vec{\boldsymbol{X}}_{\boldsymbol{c}} \prec \vec{\boldsymbol{X}}_{\boldsymbol{b}}\right) \textbf{OR} \left(\left(\vec{\boldsymbol{X}}_{\boldsymbol{c}} \,||\, \vec{\boldsymbol{X}}_{\boldsymbol{b}}\right) \textbf{ and } \left(\boldsymbol{CD}\left(\vec{\boldsymbol{X}}_{\boldsymbol{c}}\right) < \boldsymbol{CD}\left(\vec{\boldsymbol{X}}_{\boldsymbol{b}}\right)\right)\right)\right)$$

$$\vec{\boldsymbol{X}}_{\boldsymbol{d}} = \left(\vec{\boldsymbol{X}}_{\boldsymbol{c}} - \vec{\boldsymbol{X}}_{\boldsymbol{b}}\right)$$

}Else{

$$\vec{\boldsymbol{X}}_{\boldsymbol{d}} = \left(\vec{\boldsymbol{X}}_{\boldsymbol{b}} - \vec{\boldsymbol{X}}_{\boldsymbol{c}}\right)$$

}

If (*rand*(0,1) <*p*){ //change sign of *F*

$$\vec{\boldsymbol{V}_t} = \vec{\boldsymbol{X}}_{\boldsymbol{deputy}} - \boldsymbol{F} \times \vec{\boldsymbol{X}}_{\boldsymbol{d}}$$

}else{

$$\vec{\boldsymbol{V}_t} = \vec{\boldsymbol{X}}_{\boldsymbol{deputy}} + \boldsymbol{F} \times \vec{\boldsymbol{X}}_{\boldsymbol{d}}$$

}

And $\boldsymbol{i} \neq \textbf{deputy} \neq \boldsymbol{a} \neq \boldsymbol{b}$

      (b) Evaluate the candidate solution.
      (c) If the candidate dominates the parent, the candidate replaces the parent.
      If the parent dominates the candidate, the candidate is discarded. Otherwise, the candidate is added in the population.
   2.2. If the population has more than *popSize* individuals, truncate it using dominance ranking and crowding distance.

IV. EXPERIMENTS SETTINGS

To assess the performance of the proposed MODEL algorithm, it is compared to seven state-of-the-art metaheuristics using 22 bi-objective benchmark problems. This section describes the compared algorithms and their respective parameter settings, the utilized benchmark problems, and the utilized evaluation measures and stopping conditions.

A. Compared MO Metaheuristics

The proposed MODEL algorithm is compared with seven state-of-the-art MOO metaheuristics which are described in this subsection. The implmentation of thse state-of-the-art algorithms is available in the jMetal [16] multi-objective optimization framework which has been used for conducting all the experiments in this paper.

The Non-dominated Sorting Genetic Algorithm (NSGA-II) was proposed by Deb et al. [12] in 2002. This genetic algorithm consists of generating new populations from the original population by the use of classical genetic operators such as selection, crossover, and mutation. The individuals of the two populations are sorted according to their ranking. Then, the best solutions are recombined for the generation of the next population. In the case of having solutions with the same rank, a density estimation (crowding distance) is calculated with regards to the surrounding solutions for the selection of the most promising solutions.

The Strength Pareto Evolutionary Algorithm (SPEA2) was proposed by Zitzler et al. [17] in 2002. In this MOEA, every candidate solution has a fitness value which equals the sum of its strength raw fitness (solutions that dominates it) plus a density estimation. SPEA2 uses the selection, crossover, and mutation operators for generating an archive of individuals. The non-dominated solutions of both the original population and the archive are copied into a new population. In case the number of non-dominated solutions is superior to the population size, a truncation operator is used by calculating the distances among solutions. The most similar solutions are removed.

The Speed Constrained Particle Swarm Optimization (SMPSO) algorithm, which is the parent algorithm of LSMPSO, was proposed by Nebro et al. [18] in 2009. It is a particle swarm optimization algorithm for solving MOO problems. This approach is based on OMOPSO [19], whose main features are the use of the crowding distance concept adopted by NSGA-II for filtering leader solutions that are stored in an archive, the use of mutation operators for swarm speed convergence acceleration, and the use of ϵ-Dominace when generating new candidate solutions. Its main difference with respect to OMOPSO is that SMPSO incorporates a mechanism for velocity limitation and introduces a polynomial mutation operator.

The third version of the Generalized Differential Evolution algorithm (GDE3) was proposed by Kukkonen and Lampinen [6]. GDE3 is an improved version of the GDE algorithm [20], which was originally proposed in 2005. It starts with a random solution population. In every iteration, a new offspring population is generated using the differential evolution operator. Both populations are combined; then, the size of the population is reduced using non-dominated sorting and a pruning algorithm for diversity preservation as in NSGA-II. However, the GDE3 pruning algorithm modifies the NSGA-II crowding distance in order to solve some GDE3 drawbacks when dealing with problems with more than two objectives.

The cellular genetic algorithm (MOCell) was introduced by Nebro et al. [21] in 2006. Being a genetic algorithm, it uses selection, crossover, and mutation operators. Similar to many multi-objective metaheuristics, it includes an external archive for storing the non-dominated solutions discovered so far. This archive is bounded by using NSGA-II's crowding distance in order to maintain diversity in the Pareto front. The selection is achieved by selecting a solution from the neighborhood of the current solution (called cell in cGAs) and another solution selected randomly from the archive. Then, the genetic crossover and mutation operators are applied for generating a new offspring which is compared to the current offspring. If the offspring is better, it replaces the current one. Otherwise, if both solutions are non-dominated, then the worst solution in the neighborhood is replaced by the current one and inserted into the archive.

AbYSS was introduced by Nebro et al. [22] in 2008; it is a multi-objective version of a *scatter search.* It has an external archive similar to MoCell. AbYSS uses evolutionary operators such as polynomial mutation, binary crossover and solution combination.

Table 1: Utilized bi-objective problems for comparison

Problem	Number Variables	Geometries
ZDT1	30	Convex
ZDT2	30	Concave
ZDT3	30	Convex, disconnected
ZDT4	10	Convex
ZDT6	10	Concave, non-uniformly spaced
DTLZ1	7	Linear
DTLZ2	12	Concave
DTLZ3	12	Concave
DTLZ4	12	Concave
DTLZ5	12	Concave
DTLZ6	12	Concave
DTLZ7	22	Disconnected
WFG1	6	Mixed Convex-concave
WFG2	6	Convex, disconnected
WFG3	6	Linear
WFG4	6	Concave
WFG5	6	Concave
WFG6	6	Concave
WFG7	6	Concave
WFG8	6	Concave
WFG9	6	Concave

The Multi-objective Evolutionary Algorithm based on Decomposition (MOEA/D) [23] was proposed in 2007 and it consists of decomposing a MOO problem into scalar sub-problems which are optimized in parallel. Each sub-problem is transformed into a scalar aggregation problem and optimized using only neighborhood information. These neighborhood relations are determined by the calculation of distances among coefficient vectors.

B. *Benchmark Problems*

As shown in Table 1, three well-known families of problems are used for comparison purposes: the Zitzler-Deb-Thiele (ZDT) [24], the Deb-Thiele-Laumanns-Zitzler (DTLZ) [25], and the Walking-Fish-Group (WFG) family problems. These three families are the most commonly used families. They are composed of different PF geometries, namely, convex, concave, disconnected, linear, non-uniformly spaced, mixed, and linear shapes.

C. *Parameters Settings*

The parameter settings are the same for every MO metaheuristic. These parameter settings were taken from [7].

NSGA-II, SPEA2, MOCell, AbYSS and GDE3 and MOEA/D have a population size of 100. In the same manner, LSMPSO and SMPSO have a configuration of 100 particles. The metaheuristics having an archive such as NSGA-II, SPEA2 and others have also a maximum size of 100.

In regards, to the number of deputies selected for MODEL, it is fixed to 0.1. In other words, only 10% of the maximum archive size is used as deputes leaders.

The configuration parameters of the algorithms are shown in Table 2.

Table 2: Parameterization

NSGA-II	
Population size	100 Individuals
Selection of parents	Binary tournament + binary tournament
Recombination	Simulated binary, pc = 0.9
Mutation	Polynomial, pm = 1.0/L
SPEA2	
Population size	100 Individuals
Selection of parents	Binary tournament + binary tournament
Recombination	Simulated binary, pc = 0.9
Mutation	Polynomial, pm = 1.0/L
MOCell	
Population size	100 individuals (10 × 10)
Neighborhood	1-hop neighbors (8 surrounding solutions)
Selection of parents	Binary tournament + binary tournament
Recombination	Simulated binary, pc = 0.9
Mutation	Polynomial, p_m = 1.0/L
Archive size	100 individuals
SMPSO	
Particles	100 particles
Mutation	Polynomial
Leaders size	100 individuals
GDE3 / MODEL	
Population size	100 individuals
Recombination	Differential evolution, CR = 0.1, F = 0.5
MOEA/D	
Population size	100 individuals
Recombination	Differential evolution,
Mutation	Polynomial
AbYSS	
Population size	100 individuals
Reference set size	10 + 10
Recombination	Simulated binary, pc = 1.0
Mutation	Polynomial, p_m = 1.0/L
Archive size	100 individuals

D. *Performance Measure and Stopping Condition*

A high quality set of solutions, in a multi-objective optimization context, should be accurate and diverse. Accuracy means the solutions should be as close as possible to the Pareto Front. Diversity means the solution should be well-distributed to cover all of the Pareto Front. A popular quality indicator that takes into consideration both the accuracy of a solution set and its diversity is the inverse generational distance (IGD) [26]. IGD consists in measuring the average Euclidean distance between the true Pareto front and the approximation obtained by a multi-objective algorithm. Assume that P is a set of points representing the true Pareto front and S is a set of points representing the approximation obtained by a MOM. The average Euclidean distance in the objective space is calculated as follows:

$$IGD(S,P) = \frac{\sum_{v \in P} \mathrm{d}(\mathrm{v},s)}{|P|}, \tag{4}$$

where $\mathrm{d}(\mathrm{v},s)$ is the minimum Euclidean distance between a point v belonging the true Pareto front and a point s belonging to the approximation obtained. $|P|$ is the number of selected points used to represent the Pareto front. When the value of the IGD is lower, a better approximation is achieved.

V. RESULTS AND DISSCUSION

Due to the stochastic nature of metaheuristics, every algorithm was run 100 times independently. The IGD results are reported in Table 3. Dark grey shows the most accurate algorithm, while the lighter grey shows the second most accurate algorithm.

The Wilcoxon statistical procedure is conducted based on [27] to present results at a 0.05 significance level. However, whenever the statistical test did not pass between the two fastest algorithms, both of them were ranked first. For example, for the problems ZDT6 and DTLZ6, there was not a significant statistical difference between the two most accurate algorithms. The two most accurate MOMs were ranked first for these specific cases.

From Table 3, it can be clearly seen that the proposed MODEL algorithm outperformed all the other state-of-the-art MOMs, followed by its parent algorithm GDE3.

As shown in Table 4, MODEL is the most accurate for 12 problems (ZDT1-ZDT3, ZDT6, DTLZ6-DTLZ7, WFG1-WFG2, WFG4, WFG6 and WFG9) and second most accurate for 3 problems (DTLZ1, DTLZ3 and WFG7). Overall, MODEL is in top two positions for 15 out of 21 problems, i.e., 71.5%. The second position is occupied by GDE3 which is the most accurate for four problems (ZDT6, DTLZ1, DTLZ6, and WFG4) and the second most accurate for nine problems (ZDT1, ZDT3, DTLZ4, WFG1-WFG2, WFG5-WFG7, and WFG9). In addition, MODEL was more accurate or equivalently accurate to its parent algorithm GDE3 for 20 out 21 problems with the exception of the DTLZ1 problem, which is a linear problem. Therefore, MODEL proves to be consistently outperforming GDE3 in various complex problems. It can be seen from Table 3 that MODEL was not among the top two MOMs for the WFG3 which is also a linear problem; however, its performance was similar to GDE3. The performance of MODEL with linear problems needs to be further investigated.

The third most robust MOM is AbYSS. It is the most accurate for five problems; which were all concave problems, although there are also several other problems characterized by concavity where AbYSS was not accurate. In brief, AbYSS seems to be advantageous when dealing with certain types of MO optimization concave problems.

As shown in Figs. 4, 5, and 6, the MODEL algorithm is very consistent across various problems in terms of accuracy using limited number of function evaluations. In addition, it can be seen that MODEL accuracy was significantly better than all other MOMs with the WFG problems which are known to a more complex family of MOPs.

Table 3: IGD, mean and standard deviation for bi-objective problems. Dark grey is the algorithm having the best IGD results followed by light grey in the second position.

	MODEL	SPEA2	NSGAII	GDE3	SMPSO	AbYSS	MOCell	MOEAD
ZDT1	$1.62e-04_{8.6e-06}$	$9.74e-04_{3.0e-04}$	$6.09e-04_{1.3e-04}$	$2.23e-04_{5.2e-05}$	$2.10e-04_{9.9e-04}$	$7.75e-04_{6.8e-04}$	$4.37e-04_{9.8e-05}$	$2.69e-02_{7.1e-03}$
ZDT2	$1.80e-04_{2.3e-05}$	$2.73e-03_{1.6e-02}$	$9.84e-04_{3.0e-04}$	$3.47e-04_{1.1e-04}$	$1.52e-04_{1.1e-04}$	$7.31e-04_{1.5e-03}$	$2.28e-04_{1.9e-04}$	$4.95e-02_{8.5e-03}$
ZDT3	$3.83e-04_{6.2e-05}$	$1.35e-03_{4.9e-04}$	$7.33e-04_{1.6e-04}$	$5.71e-04_{1.2e-04}$	$3.72e-03_{5.2e-03}$	$5.23e-03_{4.5e-03}$	$7.96e-04_{6.0e-04}$	$3.42e-02_{4.9e-03}$
ZDT4	$1.62e-02_{8.3e-03}$	$4.26e-02_{2.0e-02}$	$1.76e-02_{1.2e-02}$	$3.20e-02_{1.3e-02}$	$1.54e-04_{1.1e-05}$	$1.37e-02_{1.1e-02}$	$4.59e-03_{5.4e-03}$	$4.21e-01_{2.8e-01}$
ZDT6	$1.17e-04_{7.2e-06}$	$2.08e-02_{4.5e-03}$	$1.23e-02_{2.3e-03}$	$1.17e-04_{7.8e-06}$	$1.20e-04_{1.1e-05}$	$1.22e-03_{4.5e-04}$	$1.45e-03_{3.5e-04}$	$6.98e-03_{3.8e-02}$
DTLZ1	$3.40e-04_{4.2e-05}$	$1.33e-01_{1.1e-01}$	$1.19e-01_{1.0e-01}$	$6.64e-04_{4.7e-04}$	$3.31e-04_{3.8e-05}$	$1.02e-01_{1.0e-01}$	$5.42e-02_{8.9e-02}$	$1.72e-01_{4.7e-01}$
DTLZ2	$3.49e-04_{9.4e-06}$	$3.58e-04_{1.3e-05}$	$4.43e-04_{3.4e-05}$	$3.51e-04_{9.1e-06}$	$3.62e-04_{1.8e-05}$	$3.44e-04_{6.9e-06}$	$3.46e-04_{8.6e-06}$	$3.58e-04_{7.0e-06}$
DTLZ3	$5.78e-01_{3.0e-01}$	$1.42e+00_{7.7e-01}$	$1.21e+00_{6.6e-01}$	$8.61e-01_{2.9e-01}$	$8.46e-04_{3.1e-02}$	$1.51e+00_{9.1e-01}$	$7.85e-01_{5.6e-01}$	$1.49e+00_{2.9e+00}$
DTLZ4	$1.66e-04_{4.6e-05}$	$1.11e-04_{6.6e-03}$	$1.11e-04_{2.6e-05}$	$9.60e-05_{2.0e-05}$	$1.02e-04_{2.5e-05}$	$8.69e-05_{7.4e-06}$	$6.70e-03_{6.6e-03}$	$3.28e-04_{1.4e-04}$
DTLZ5	$3.48e-04_{7.8e-06}$	$3.58e-04_{1.1e-05}$	$4.45e-04_{2.5e-05}$	$3.53e-04_{1.1e-05}$	$3.58e-04_{2.1e-05}$	$3.43e-04_{7.5e-06}$	$3.46e-04_{8.8e-06}$	$3.58e-04_{7.8e-06}$
DTLZ6	$3.49e-04_{6.9e-06}$	$1.68e-01_{1.1e-02}$	$1.25e-01_{9.5e-03}$	$3.49e-04_{5.4e-06}$	$4.20e-04_{6.1e-02}$	$5.16e-02_{1.4e-02}$	$4.40e-02_{9.2e-03}$	$4.07e-04_{9.7e-04}$
DTLZ7	$2.00e-04_{1.7e-05}$	$8.78e-04_{2.8e-04}$	$4.69e-04_{9.6e-05}$	$2.22e-04_{2.3e-05}$	$1.79e-04_{1.4e-05}$	$2.88e-04_{2.0e-04}$	$2.73e-04_{5.8e-05}$	$4.25e-02_{1.4e-02}$
WFG1	$7.15e-04_{2.8e-04}$	$5.22e-03_{9.6e-04}$	$4.29e-03_{8.4e-04}$	$9.66e-04_{1.9e-04}$	$4.79e-03_{6.2e-05}$	$5.70e-03_{1.2e-03}$	$4.15e-03_{9.0e-04}$	$4.37e-03_{7.4e-04}$
WFG2	$5.32e-05_{6.1e-06}$	$3.48e-04_{2.8e-04}$	$3.48e-04_{2.8e-04}$	$5.60e-05_{7.4e-06}$	$1.44e-04_{3.8e-05}$	$3.49e-04_{4.0e-06}$	$3.48e-04_{1.5e-06}$	$2.54e-04_{4.1e-05}$
WFG3	$6.84e-04_{8.4e-08}$	$6.86e-04_{6.9e-06}$	$6.84e-04_{1.6e-06}$	$6.84e-04_{1.1e-07}$	$6.82e-04_{1.3e-06}$	$6.83e-04_{1.2e-06}$	$6.83e-04_{9.7e-07}$	$6.83e-04_{4.5e-07}$
WFG4	$9.16e-05_{2.5e-06}$	$1.15e-04_{6.5e-06}$	$1.25e-04_{1.3e-05}$	$9.21e-05_{3.0e-06}$	$3.46e-04_{3.1e-05}$	$9.19e-05_{2.6e-06}$	$9.46e-05_{3.8e-06}$	$3.86e-04_{4.0e-05}$
WFG5	$5.38e-04_{5.6e-07}$	$5.45e-04_{2.2e-06}$	$5.51e-04_{4.8e-06}$	$5.38e-04_{4.9e-07}$	$5.38e-04_{1.2e-06}$	$5.39e-04_{9.5e-07}$	$5.40e-04_{1.1e-06}$	$5.48e-04_{2.2e-06}$
WFG6	$9.56e-05_{1.3e-05}$	$1.70e-04_{1.3e-04}$	$1.93e-04_{1.5e-04}$	$9.73e-05_{1.5e-05}$	$1.29e-04_{1.7e-05}$	$6.86e-04_{7.3e-04}$	$3.92e-04_{6.4e-04}$	$1.31e-04_{9.2e-06}$
WFG7	$8.73e-05_{2.4e-06}$	$9.66e-05_{4.1e-06}$	$1.22e-04_{1.3e-05}$	$8.72e-05_{2.0e-06}$	$1.12e-04_{1.2e-05}$	$8.65e-05_{2.0e-06}$	$8.69e-05_{2.4e-06}$	$1.31e-04_{4.3e-06}$
WFG8	$1.07e-03_{2.3e-05}$	$1.06e-03_{1.2e-04}$	$9.40e-04_{1.4e-04}$	$1.07e-03_{4.1e-05}$	$1.01e-03_{6.8e-05}$	$1.07e-03_{3.7e-05}$	$1.06e-03_{3.8e-05}$	$8.95e-04_{1.6e-04}$
WFG9	$1.09e-04_{1.4e-05}$	$1.25e-04_{1.8e-05}$	$1.51e-04_{1.8e-05}$	$1.13e-04_{1.6e-05}$	$1.59e-04_{8.3e-06}$	$1.35e-04_{4.2e-05}$	$1.22e-04_{2.4e-05}$	$1.79e-04_{7.6e-06}$

Table 4: Scores for comparison (3 points for the 1st position, and 1 point for the 2nd position, Occ.=Number Occurances, Pnts=Number of points allocated)

	MODEL		NSGA-II		SPEA2		GDE3		SMPSO		AbYSS		MOCell		MOEA/D	
	Occ.	Pnts	Occ.	Pnts	Occ.	Pnts	Occ.	Pnts	Occ.	Pnts	Occ.	Pnts	Occ.	Pnts	Occ.	Pnts
1st	12	36	0	0	0	0	4	12	3	9	5	15	0	0	1	3
2nd	3	3	1	1	2	6	9	9	1	1	1	1	1	5	0	0
Total	39		1		6		21		10		16		5		3	

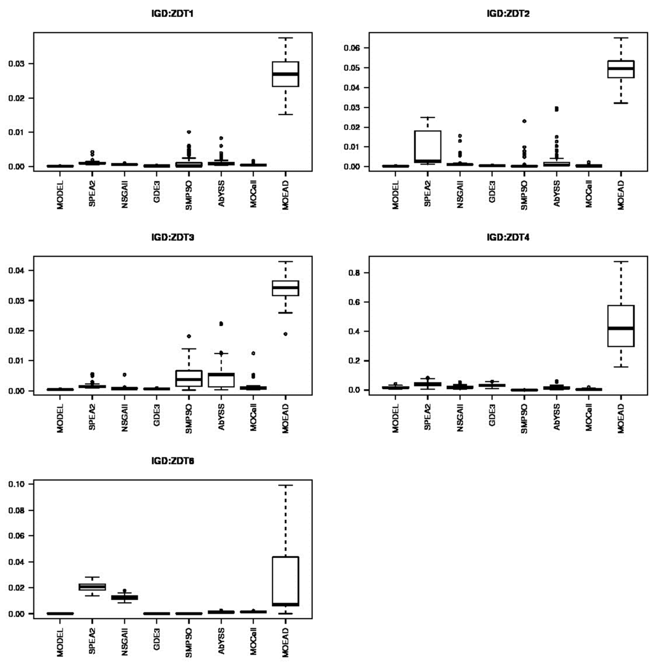

Figure 4: IGD boxplots for ZDT bi-objective problems

VI. CONCLUSIONS

This paper proposed a new MOM named MODEL based on a MO version of DE. MODEL incorporated the leadership concept into DE's mutation operator. The leadership served to accelerate GDE3 MOM using the GDE3/deputy/1 mutation scheme. In addition, the non-dominance concept was utilized to determine the leader among the two vectors in the difference vector to generate the mutant vectors. Finally, the negation of the control factor F with a probability was used to avoid premature convergence in specific cases where the distance between the initial population and the true PF was very large.

The proposed MODEL has been compared to seven state-of-the-art metaheuristics using 21 bi-objective problems taken from ZDT, DTLZ, and WFG. MODEL was in average the most accurate MOM. MODEL outperformed its parent algorithms GDE3 almost all the time. Also, Abyss showed good accuracy for certain concave problems.

In future, we would like to test the proposed method on many-objective problems. It is also important to examine the performance of MODEL by varying the configuration parameters.

To demonstrate the performance of the proposed MODEL MOM, it would be useful to apply the MODEL algorithm to real case studies.

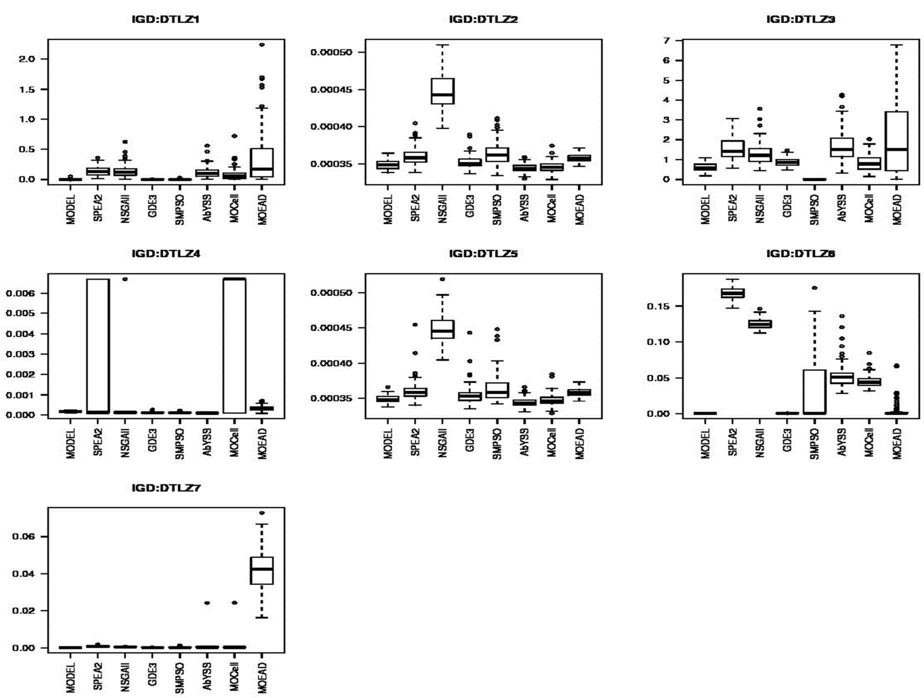

Figure 5: IGD boxplots for DTLZ bi-objective problems

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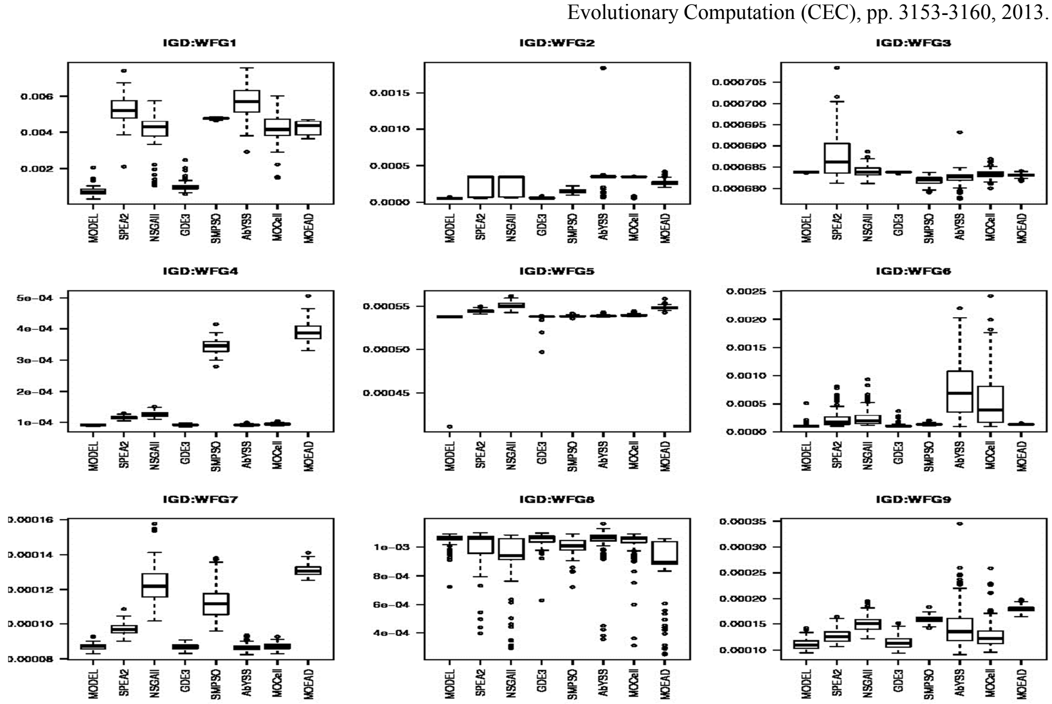

Figure 6: IGD boxplots for WFG bi-objective problem